

## 11.4 Geometric Sequences

A geometric sequence is a sequence in which the ratio of any term to the previous term is **constant**. This constant ratio is called the **common ratio** and is denoted by  $r$ .

Decide whether each sequence is geometric.

If so, identify the common ratio and give the next 3 terms.

1. 1, 2, 6, 24, 120, ...

not geometric

2. 81, 27, 9, 3, 1, ...

geometric

$$r = \frac{1}{3}$$

3. 4, -8, 16, -32, ...

geometric

$$r = -2$$

4. 3, 9, -27, -81, 243, ...

not geometric

List the first four terms of each geometric sequence.

5.  $t_1 = -2$

$$t_n = 4t_{n-1}$$

$$t_2 = 4t_1$$

$$t_2 = 4(-2) = -8$$

$$t_3 = 4t_2$$

$$t_3 = 4(-8) = -32$$

$$t_4 = 4t_3$$

$$t_4 = 4(-32) = -128$$

6.  $t_1 = -1$

$$t_n = -0.2t_{n-1}$$

$$t_2 = -0.2t_1$$

$$t_2 = -0.2(-1) = 0.2$$

$$t_3 = -0.2t_2$$

$$t_3 = -0.2(0.2) = -0.04$$

$$t_4 = -0.2t_3$$

$$t_4 = -0.2(-0.04) = 0.008$$

## $n$ th Term of a Geometric Sequence

$$t_n = t_1 r^{n-1}$$

7. Find the eighth term of the sequence defined

by  $t_1 = 2.5$  and  $t_n = -4 t_{n-1}$ .

$$t_8 = 2.5(-4)^{8-1}$$

$$t_8 = -40,960$$

8. Find  $t_9$  of a geometric sequence if  $t_1 = 144$  and  $r = \frac{1}{2}$ .

$$t_9 = 144 \left(\frac{1}{2}\right)^{9-1}$$

$$t_9 = \frac{9}{16}$$

9. Find  $t_6$  in the geometric sequence that includes

$t_3 = 150$  and  $t_5 = 3750$ .

$$n=5$$

$$k=3$$

$$t_n = t_1 r^{n-1}$$

$$t_n = t_k r^{n-k}$$

$$t_5 = t_3 r^{5-3}$$

$$\frac{3750}{150} = \frac{150 r^2}{150}$$

$$\sqrt{25} = \sqrt{r^2}$$

$$\pm 5 = r$$

$$t_6 = t_3 r^{6-3}$$

$$t_6 = 150(5)^3$$

$$t_6 = 18750$$

$$t_6 = 150(-5)^3$$

$$t_6 = -18,750$$

10. Find  $t_6$  in the geometric sequence that includes

$$t_4 = 36 \text{ and } t_8 = 2916,$$

$$t_n = t_k r^{n-k}$$

$$t_8 = t_4 r^{8-4}$$

$$\frac{2916}{36} = \frac{36r^4}{36}$$

$$\sqrt[4]{81} = \sqrt[4]{r^4}$$

$$\pm 3 = r$$

$$n=8 \quad k=4$$

$$t_6 = t_4 r^{6-4}$$

$$t_6 = 36(3)^2$$

$$t_6 = 324$$

$$t_6 = 36(-3)^2$$

11. Write an explicit formula for the  $n$ th term of the geometric sequence  $30, 10, 3\frac{1}{3}, 1\frac{1}{9}, \dots$   $r = \frac{1}{3}$

$$t_n = t_1 r^{n-1}$$

$$t_n = 30\left(\frac{1}{3}\right)^{n-1}$$

The terms between any two nonconsecutive terms of a geometric sequence are called the geometric means.

12. Find the three geometric means between 6 and 96.

$$6 \quad \underline{\pm 12} \quad \underline{24} \quad \underline{\pm 48} \quad 96$$

$\swarrow$   $\searrow$     $\swarrow$   $\searrow$     $\swarrow$   $\searrow$     $\swarrow$   $\searrow$   
 $\times r$     $\times r$     $\times r$     $\times r$

$$\frac{6r^4}{6} = \frac{96}{6}$$

$$\sqrt[4]{r^4} = \sqrt[4]{16}$$

$$r = \pm 2$$

13. Find the three geometric means between 324 and 4.

$$324 \quad \underline{\pm 108} \quad \underline{36} \quad \underline{\pm 12} \quad 4$$

$\swarrow$   $\searrow$     $\swarrow$   $\searrow$     $\swarrow$   $\searrow$     $\swarrow$   $\searrow$   
 $\times r$     $\times r$     $\times r$     $\times r$

$$\frac{324r^4}{324} = \frac{4}{324}$$

$$\sqrt[4]{r^4} = \sqrt[4]{\frac{1}{81}}$$

$$r = \pm \frac{1}{3}$$