

## 11.2 ARITHMETIC SEQUENCES

In an arithmetic sequence, the difference between consecutive terms is constant. The constant difference is called the common difference and is denoted by  $d$ .

**Examples:** Decide whether each sequence is arithmetic. If so, identify the common difference.

1.  $-7, -3, 1, 5, 9, \dots$  *arithmetic,  $d=4$*
2.  $2, 3, 5, 8, 12, 17, \dots$  *not arithmetic*
3.  $-14, -8, -2, 0, 2, 8, 14, \dots$  *not arithmetic*
4.  $19, 13, 7, 1, -5, \dots$  *arithmetic,  $d=-6$*

5. List the first four terms of each arithmetic sequence.

a)  $t_n = -3 + (n - 1)(5)$

$$t_1 = -3 + \cancel{(1-1)}(5)$$

$$t_1 = -3$$

$$t_2 = -3 + \cancel{(2-1)}(5)$$

$$t_2 = 2$$

$$t_3 = -3 + \cancel{(3-1)}(5)$$

$$t_3 = 7$$

$$t_4 = -3 + \cancel{(4-1)}(5)$$

$$t_4 = 12$$

b)  $t_n = \pi n + 4$

$$t_1 = \pi \cdot 1 + 4$$

$$t_1 = \pi + 4$$

$$t_2 = \pi \cdot 2 + 4$$

$$t_2 = 2\pi + 4$$

$$t_3 = \pi \cdot 3 + 4$$

$$t_3 = 3\pi + 4$$

$$t_4 = \pi \cdot 4 + 4$$

$$t_4 = 4\pi + 4$$

The  $n$ th term of an arithmetic sequence with first term  $t_1$  and common difference  $d$  is given by:

$$t_n = t_1 + (n - 1)d$$

↓  
first term

6. Write an explicit formula for the  $n$ th term of the arithmetic sequence 50, 44, 38, 32, ...  $d = -6$

$$t_n = 50 + (n - 1)(-6)$$

$$t_n = 50 - 6n + 6$$

$$t_n = 56 - 6n$$

7. Write an explicit formula for the  $n$ th term of the arithmetic sequence shown below. Then find the 20th term.  $n = 20$

32, 47, 62, 77, ...  $d = 15$

$$t_n = t_1 + (n - 1)d$$

$$t_n = 32 + (n - 1) \cdot 15$$

$$t_n = 32 + 15n - 15$$

$$t_n = 17 + 15n$$

$$t_{20} = 17 + 15(20)$$

$$t_{20} = 317$$

8. Find the 40th term of the sequence defined by  $t_1 = 7$  and  $t_n = t_{n-1} + 6$ . ← recursive formula

$$t_2 = t_1 + 6$$

$$t_2 = 7 + 6$$

$$t_2 = 13$$

$$d = 6$$

$$t_{40} = t_1 + (n-1)d$$

$$t_{40} = 7 + (40-1)(6)$$

$$t_{40} = 7 + 240 - 6$$

$$t_{40} = 241$$

9. Find the 101st term of the sequence defined by  $t_1 = -4$  and  $t_n = t_{n-1} - 2$ .  $d = -2$

$$t_n = t_1 + (n-1)d$$

$$t_{101} = -4 + (101-1)(-2)$$

$$t_{101} = -4 + -200$$

$$t_{101} = -204$$

10. Find the 10th term of the arithmetic sequence in which  $t_3 = -5$  and  $t_6 = 16$ .

$$t_n = t_1 + (n-1)d$$

$$\rightarrow t_n = t_k + (n-k)d$$

$$t_6 = t_3 + (6-3)d$$

$$16 = -5 + 3d$$

$$21 = 3d$$

$$7 = d$$

$$t_{10} = t_3 + (10-3)7$$

$$t_{10} = -5 + 49$$

$$t_{10} = 44$$

11. Find the 15th term of the arithmetic sequence in which  $t_5 = 7$  and  $t_{10} = 22$ .

$$t_n = t_k + (n-k)d$$

$$t_{10} = t_5 + (10-5)d$$

$$22 = 7 + 5d$$

$$15 = 5d$$

$$3 = d$$

$$t_{15} = t_5 + (15-5) \cdot 3$$

$$t_{15} = 7 + 30$$

$$t_{15} = 37$$

The terms between any two nonconsecutive terms of an arithmetic sequence are called the arithmetic means.

12. Find the four arithmetic means between 10 and -30.

$$10 \quad 2 \quad -6 \quad -14 \quad -22 \quad -30$$

$+d \quad +d \quad +d \quad +d \quad +d$

$$10 + 5d = -30$$

$$5d = -40$$

$$d = -8$$

13. Find the five arithmetic means between 6 and 60.

$$6 \quad 15 \quad 24 \quad 33 \quad 42 \quad 51 \quad 60$$

$+d \quad +d \quad +d \quad +d \quad +d \quad +d$

$$6 + 6d = 60$$

$$6d = 54$$

$$d = 9$$