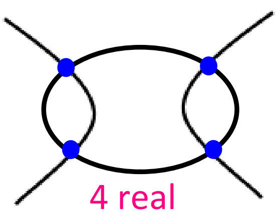
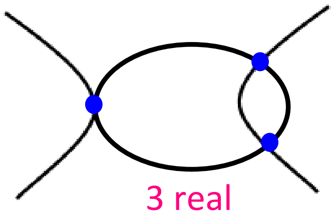
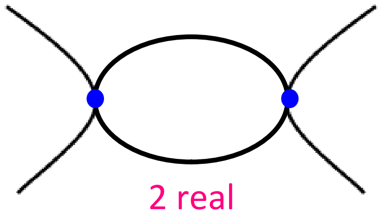
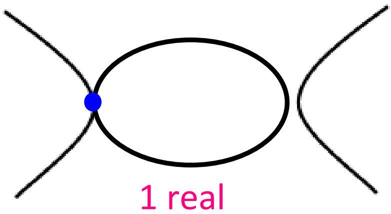
9.6 PART 2 Solving Nonlinear Systems

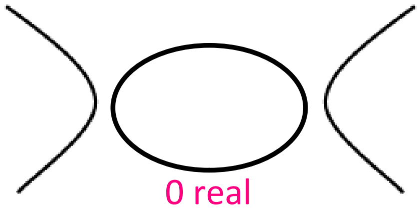
A **system of nonlinear equations** is a collection of equations in which at least one equation is not linear.


4 real solutions


3 real solutions


2 real solutions


1 real solution


0 real solutions

Example 1

Solve $\begin{cases} y^2 = \boxed{3x - 1} \\ x^2 + y^2 = 9 \end{cases}$ by substitution.

If there are no real solutions, write none.

$x^2 + 3x - 1 = 9$

$x^2 + 3x - 10 = 0$

$(x+5)(x-2) = 0$

$x = -5 \quad x = 2$

$(2, \sqrt{5}), (2, -\sqrt{5})$

$x = -5$

$y^2 = 3(-5) - 1$

$\cancel{\sqrt{y^2} = \sqrt{-16}}$

$x = 2$

$y^2 = 3(2) - 1$

$\sqrt{y^2} = \sqrt{5}$

$y = \pm\sqrt{5}$

Example 2

Solve $\begin{cases} y^2 = \boxed{4x + 13} \\ x^2 + y^2 = 25 \end{cases}$ by substitution.

If there are no real solutions, write none.

$x^2 + 4x + 13 = 25$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x = -6 \quad x = 2$

$(2, \sqrt{21}), (2, -\sqrt{21})$

$x = -6$

$y^2 = 4(-6) + 13$

$\cancel{\sqrt{y^2} = \sqrt{-11}}$

$x = 2$

$y^2 = 4(2) + 13$

$\sqrt{y^2} = \sqrt{21}$

$y = \pm\sqrt{21}$

Example 3

Solve $\begin{cases} x^2 + y^2 = 25 \\ y^2 = 2x + 1 \end{cases}$ by substitution.

If there are no real solutions, write none.

$x^2 + 2x + 1 = 25$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

$x = -6 \quad x = 4$

$(4, 3), (4, -3)$

$x = -6$

$\cancel{y^2 = 2(-6) + 1}$

$\cancel{\sqrt{y^2} = \sqrt{-11}}$

$x = 4$

$y^2 = 2(4) + 1$

$y^2 = 9$

$y = \pm 3$

Example 4

Solve $\begin{cases} y = \boxed{x} \\ y^2 - x^2 = 8 \end{cases}$ by substitution.

If there are no real solutions, write none.

$(x)^2 - x^2 = 8$

$0 \neq 8$

no solution

Example 5

Solve $\begin{cases} -x^2 - \cancel{y^2} = -9 \\ 9x^2 + \cancel{y^2} = 9 \end{cases}$ by elimination.

If there are no real solutions, write none.

$\frac{8x^2}{8} = \frac{0}{8}$

$x^2 = 0$

$x = 0$

$(0, 3), (0, -3)$

$0^2 + y^2 = 9$

$y^2 = 9$

$y = \pm 3$

Example 6

Solve $\begin{cases} x^2 - y^2 = 36 \\ 4y^2 - 9x^2 = 36 \end{cases}$ by elimination.

If there are no real solutions, write none.

$4(x^2 - y^2) = (36)4$

$-9x^2 + \cancel{4y^2} = 36$

$4x^2 - \cancel{4y^2} = 144$

$-5x^2 = 180$

$\frac{-5x^2}{-5} = \frac{180}{-5}$

$\sqrt{x^2} = \sqrt{-36}$

no solution

Example 7

Solve $\begin{cases} -4(x^2 + y^2) = (36) \cdot -4 \\ 4x^2 - 9y^2 = 36 \end{cases}$ by elimination.

If there are no real solutions, write none.

$\cancel{-4x^2} - 4y^2 = -144$

$\cancel{4x^2} - 9y^2 = 36$

$\frac{-13y^2}{-13} = \frac{-108}{-13}$

$\sqrt{y^2} = \sqrt{\frac{108}{13}}$

$y = \pm\frac{6\sqrt{3}}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$

$y = \pm\frac{6\sqrt{39}}{13}$

$x^2 + \left(\pm\frac{6\sqrt{39}}{13}\right)^2 = 36$

$x^2 + \frac{108}{13} = 36$

$-\frac{108}{13} \quad -\frac{108}{13}$

$\sqrt{x^2} = \sqrt{\frac{360}{13}}$

$x = \pm\frac{6\sqrt{10}}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}$

$x = \pm\frac{6\sqrt{130}}{13}$

$\left(\frac{6\sqrt{130}}{13}, \frac{6\sqrt{39}}{13}\right), \left(\frac{6\sqrt{130}}{13}, -\frac{6\sqrt{39}}{13}\right), \left(-\frac{6\sqrt{130}}{13}, \frac{6\sqrt{39}}{13}\right), \left(-\frac{6\sqrt{130}}{13}, -\frac{6\sqrt{39}}{13}\right)$