

9.6 PART 1 Classifying a Conic Section

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

Type of Conic	Coefficients
Ellipse	$AC > 0$
Circle	$A = C, A \neq 0, C \neq 0$
Parabola	$AC = 0$
Hyperbola	$AC < 0$

Example 1

Let $4x^2 + 8x = y^2 + 6y + 13$ be the equation of a conic section. Classify the conic section and write the standard equation.

$$4x^2 + 8x - y^2 - 6y = 13$$

$$4(x^2 + 2x + 1) - (y^2 + 6y + 9) = 13 + 4 - 9$$

$\frac{1}{2}(2) = 1$
 $(1)^2 = 1$

$\frac{1}{2}(6) = 3$
 $(3)^2 = 9$

$$\frac{4(x+1)^2}{8} - \frac{(y+3)^2}{8} = \frac{8}{8}$$

$$\frac{(x+1)^2}{2} - \frac{(y+3)^2}{8} = 1 \rightarrow \text{hyperbola}$$

Example 2

Let $6x^2 + 12x = -3y - 9$ be the equation of a conic section. Classify the conic section and write the standard equation.

$$6(x^2 + 2x + 1) = -3y - 9 + 6$$

$$\frac{1}{2}(2) = 1$$

$$(1)^2 = 1$$

$$6(x+1)^2 = -3y - 3$$

$$\frac{6(x+1)^2 + 3}{-3} = \frac{-3y}{-3}$$

$$-2(x+1)^2 - 1 = y \rightarrow \text{parabola}$$

Example 3

Let $9x^2 + 18x + 4y^2 + 8y = 23$ be the equation of a conic section. Classify the conic section and write the standard equation.

$$9(x^2 + 2x + 1) + 4(y^2 + 2y + 1) = 23 + 9 + 4$$

$$\frac{9(x+1)^2}{36} + \frac{4(y+1)^2}{36} = \frac{36}{36}$$

$$\frac{(x+1)^2}{4} + \frac{(y+1)^2}{9} = 1 \rightarrow \text{ellipse}$$