# 7.5 More Trigonometric Equations

Sometimes, we use our identities to help solve.
It is helpful if the problem uses only one trigonometric function.

## Example 1: Solve.

$1 + \sin x = 2\cos^2 x$

$1 + \sin x = 2(1 - \sin^2 x)$

$\sin x = y$

$1 + \sin x = 2 - 2\sin^2 x$

$-2 + 2\sin^2 x \quad -2 + 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$2y^2 + y - 1 = 0$

$(y+1)(2y-1) = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x + 1 = 0 \qquad 2\sin x - 1 = 0$

$\sin x = -1 \qquad \sin x = \frac{1}{2}$

$x = \frac{3\pi}{2} \pm 2\pi k \qquad x = \frac{\pi}{6} \pm 2\pi k$

$x = \frac{5\pi}{6} \pm 2\pi k$

## Example 2: Solve.

$\sin 2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x\,(2\sin x - 1) = 0$

$\cos x = 0 \qquad 2\sin x - 1 = 0$

$x = \frac{\pi}{2} \pm \pi k \qquad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6} \pm 2\pi k$

$x = \frac{5\pi}{6} \pm 2\pi k$

Example 3: Solve the equation below in the interval $[0,2\pi)$.

$$(\cos x + 1)^2 = (\sin x)^2$$

hint: square both sides...which means you'll have to check for extraneous solutions

$\cos^2 x + 2\cos x + 1 = \sin^2 x$

$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$

$+\cos^2 x \qquad -1 \quad -1 + \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$2\cos x = 0 \qquad \cos x + 1 = 0$

$\cos x = 0 \qquad \cos x = -1$

$x = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}} \qquad x = \pi$

$\cos\frac{\pi}{2} + 1 \stackrel{?}{=} \sin\frac{\pi}{2}$

$0 + 1 = 1$ ✓

$\cos\frac{3\pi}{2} + 1 \stackrel{?}{=} \sin\frac{3\pi}{2}$

$0 + 1 \neq -1$

$\cos\pi + 1 \stackrel{?}{=} \sin\pi$

$-1 + 1 = 0$ ✓

When solving trigonometric equations that involve functions of multiples of angles, we first solve for the multiple of the angle, then divide to solve for the angle.

Example 4: Solve the equation below in the interval $[0,2\pi)$.

$$2\sin 3x - 1 = 0$$

$\sin 3x = \frac{1}{2}$

$\frac{1}{3}\cdot 3x = \frac{\pi}{6}\cdot\frac{1}{3} \pm 2\pi k \qquad \frac{1}{3}\cdot 3x = \frac{5\pi}{6}\cdot\frac{1}{3} \pm 2\pi k$

$x = \frac{\pi}{18} \pm \frac{2}{3}\pi k \qquad x = \frac{5\pi}{18} \pm \frac{2\pi}{3}k$

$x = \frac{\pi}{18}, \frac{13\pi}{18}, \frac{25\pi}{18} \qquad x = \frac{5\pi}{18}, \frac{17\pi}{18}, \frac{29\pi}{18}$

Example 5: Solve the equation below in the interval $[0,2\pi)$.

$$\sqrt{3}\tan\frac{x}{2} - 1 = 0$$

$$\tan\frac{x}{2} = \frac{\sqrt{3}}{3}$$

$$2\cdot\frac{x}{2} = \left(\frac{\pi}{6} + \pi k\right)\cdot 2$$

$$x = \frac{\pi}{3} + 2\pi k$$

$$x = \frac{\pi}{3}$$

Example 6: Find all solutions of the equation.

$$\sin 2x = 2\tan 2x$$

$$\cos 2x \cdot \sin 2x = \frac{2\sin 2x}{\cancel{\cos 2x}} \cdot \cos 2x$$

$$\sin 2x \cos 2x = 2\sin 2x$$

$$\sin 2x\cos 2x - 2\sin 2x = 0$$

$$\sin 2x\,(\cos 2x - 2) = 0$$

$$\sin 2x = 0 \qquad \cos 2x - 2 = 0$$

$$\frac{2x}{2} = \frac{\pi k}{2} \qquad \cos 2x \cancel{=} 2$$

$$x = \frac{\pi k}{2}$$