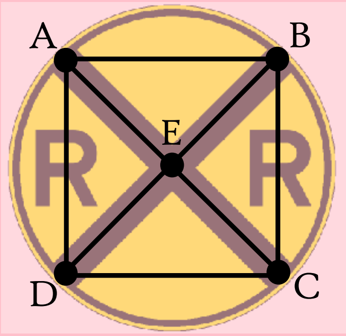
10.3 ARCS & CHORDS

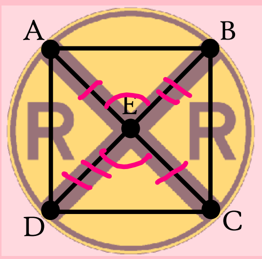
When a minor arc and a chord share the same endpoints, we call the arc the arc of the chord.



In the railroad crossing sign, $\overset{\frown}{AB}$ is the arc of $\overline{AB}$.

Theorem 10.4

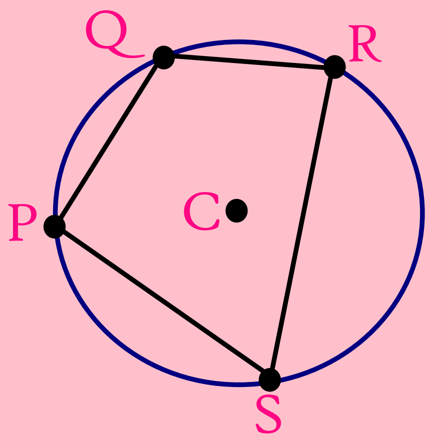
In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



Statements	Reasons
1. $\odot E$, $\overset{\frown}{AB} \cong \overset{\frown}{DC}$	① given
2. $\overline{AE} \cong \overline{CE}$ $\overline{BE} \cong \overline{DE}$	② radii in same circle are $\cong$
3. $\angle AEB \cong \angle CED$	③ vert $\angle$s $\cong$
4. $\triangle AEB \cong \triangle CED$	④ SAS
5. $\overline{AB} \cong \overline{DC}$	⑤ CPCTC

A polygon is an <u>inscribed polygon</u> if each of its vertices lies on a circle.

Quadrilateral PQRS is inscribed in ⊙C.



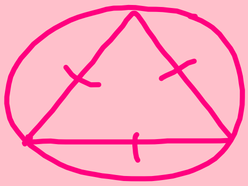
<u>Example 1</u>

A stop sign is an octagon with congruent sides. It can be inscribed in a circle by using the center of the sign and a vertex of the sign as endpoints for the radius of the circle, as in QR. Find the measure of each of the right corresponding arcs of a circle around the stop sign shown below.

$\frac{360}{8} = \boxed{45}$

Example 2

a) Find the measure of each minor arc created when an equilateral triangle is inscribed in a circle.



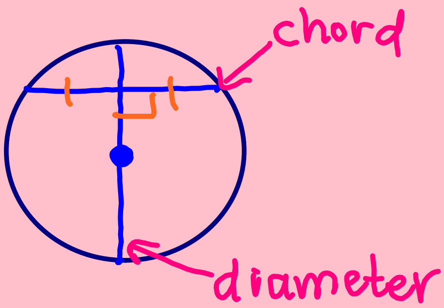
$$\frac{360}{3} = 120^\circ$$

b) Find the measure of each minor arc created when a regular dodecagon is inscribed in a circle.

$$\frac{360}{12} = 30^\circ$$

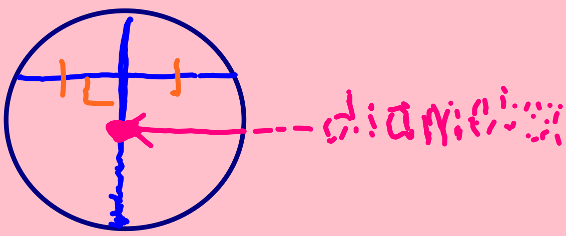
Theorem 10.5

In a circle, if a diameter is perpendicular to a chord, then it bisects the chord and its arc.



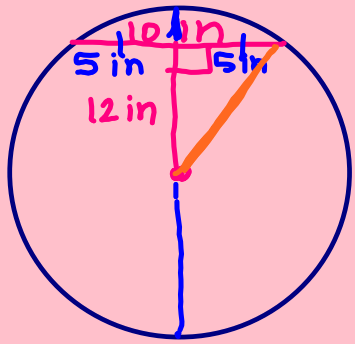
Theorem 10.6

If one chord is a perpendicular bisector of another chord, then the first chord is a diameter.



Example 3

Suppose a chord of a circle is 10 inches long and 12 inches from the center of the circle. Find the length of the radius.

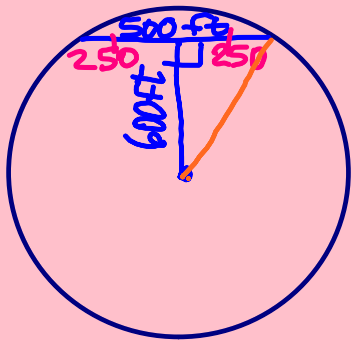


$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$169 = c^2$$
$$13 = c$$

radius = 13 in

Example 4

You discovered a crop circle in a nearby farm. A chord of the circle is 500 feet long and 600 feet from the center of the circle. Find the length of the radius.

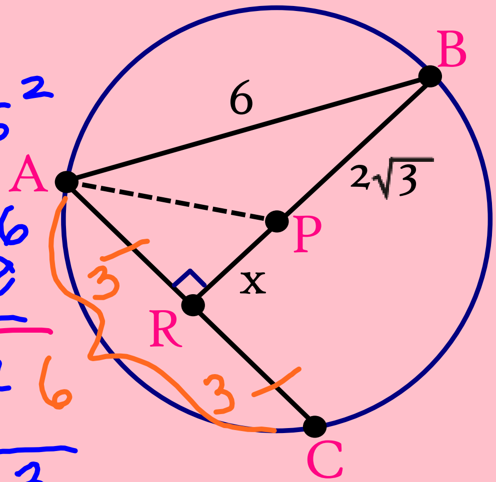


$$250^2 + 600^2 = c^2$$
$$62{,}500 + 360{,}000 = c^2$$
$$\sqrt{422{,}500} = \sqrt{c^2}$$
$$650 = c$$

radius = 650 ft

Example 5

In $\odot P$, $\overline{AB} \cong \overline{AC}$. Find the value of x as a radical.



$$3^2 + (x+2\sqrt{3})^2 = 6^2$$

$$9 + (x+2\sqrt{3})^2 = 36$$

$$-9 \qquad -9$$

$$\sqrt{(x+2\sqrt{3})^2} = \sqrt{27}$$

$$x + 2\sqrt{3} = 3\sqrt{3}$$

$$-2\sqrt{3} \qquad -2\sqrt{3}$$

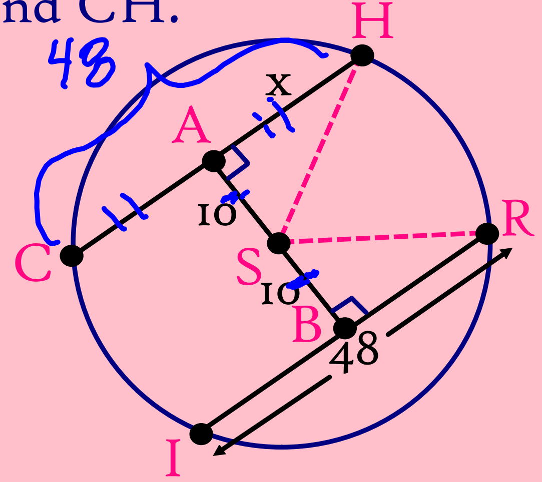
$$x = \sqrt{3}$$

Theorem 10.7

In a circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

Example 6

Chords $\overline{CH}$ are $\overline{IR}$ are equidistant from the center of $\odot S$. If IR = 48, find CH.



$$CH = 48$$