

7.4 Trigonometric Equations

To solve a trigonometric equation, we must find **all** the values of the variable that make the equation true. Use the rules of algebra to isolate the trigonometric function on one side of the equal sign. Then, use your knowledge of the values of trigonometric functions to solve for the variable.

Example 1: Solve.

$$2 \sin x - 1 = 0$$

$$\begin{array}{r} +1 \quad +1 \\ \hline 2 \sin x = 1 \\ \hline \frac{2 \sin x}{2} = \frac{1}{2} \end{array}$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} \pm 2\pi k, \frac{5\pi}{6} \pm 2\pi k$$

Example 2: Solve.

$$\tan^2 x - 3 = 0$$

$$\begin{array}{r} +3 \quad +3 \\ \hline \sqrt{\tan^2 x} = \sqrt{3} \end{array}$$

$$\tan x = \pm \sqrt{3}$$

$$x = \frac{\pi}{3} + \pi k, \frac{2\pi}{3} + \pi k$$

QI, QIII

QII, QIV

Example 3: Solve.

$$\frac{\sin x}{\cos x} = 1$$

$$\tan x = 1$$

$$x = \frac{\pi}{4} \pm \pi k$$

Example 4: Solve.

$$\cos^{-1}(\cos x) = \cos^{-1}(0.32)$$

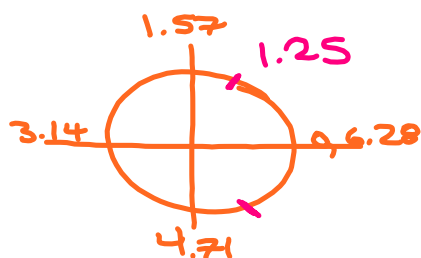
$$x = 1.25 + 2\pi k, 5.03 \pm 2\pi k$$

QI

QIV

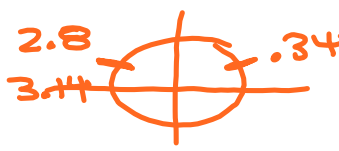
or

$$-1.25 \pm 2\pi k$$



Example 5: Solve.

$$9 \sin^2 x - 1 = 0$$



$$\frac{9 \sin^2 x}{9} = \frac{1}{9}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{1}{9}}$$

$$\sin^{-1}(\sin x) = \sin^{-1}\left(\frac{1}{3}\right) \quad \sin x = \pm \frac{1}{3}$$

$$\sin x = -\frac{1}{3}$$

$$x \approx 0.34 \pm \pi k \quad \text{QI, QIII}$$

$$x \approx 2.80 \pm \pi k \quad \text{QII, QIV}$$

Factoring is one of the most useful techniques for solving trigonometric equations. Set the equation equal to 0, then factor.

$$\cos x \rightarrow y$$

Example 6: Solve.

$$2 \cos^2 x - 7 \cos x + 3 = 0$$

$$2y^2 - 7y + 3 = 0$$

$$(2y - 1)(y - 3) = 0$$

$$(2 \cos x - 1)(\cos x - 3) = 0$$

$$2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3} \pm 2\pi k$$

$$x = \frac{5\pi}{3} \pm 2\pi k$$

$$\cos x - 3 = 0$$

$$\cos x = 3$$

Example 7: Solve.

$$3 \tan^3 \theta = \tan \theta$$

$$3 \tan^3 \theta - \tan \theta = 0$$

$$\underline{\tan \theta} (3 \tan^2 \theta - 1) = 0$$

$$\tan \theta = 0$$

$$\theta = \pi k$$

$$3 \tan^2 \theta - 1 = 0$$

$$\sqrt{\tan^2 \theta} = \sqrt{\frac{1}{3}}$$

$$\tan \theta = \pm \frac{\sqrt{3}}{3}$$

$$\theta = \frac{\pi}{6} + \pi k$$

$$\theta = \frac{5\pi}{6} + \pi k$$

Example 8: Solve.

$$4 \cos x \sin x + 3 \cos x = 0$$

$$\underline{\cos x} (4 \sin x + 3) = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2} \pm \pi k$$

$$4 \sin x + 3 = 0$$

$$\sin^{-1}(\sin x) = \sin^{-1}\left(-\frac{3}{4}\right)$$

$$x \approx -0.85 \pm 2\pi k$$

$$x \approx -2.29 \pm 2\pi k$$

