CHAPTER 10 CIRCLES
10.1: Tangents to Circles

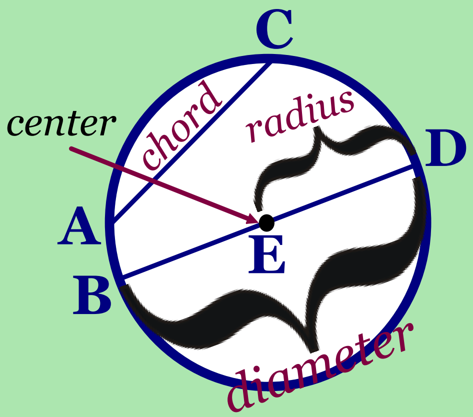
<u>circle</u>- the set of all points in a plane that are a given distance from a given point in that plane

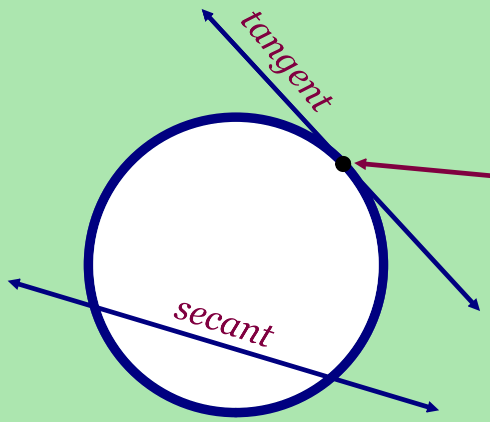
<u>center</u>- the given point

<u>radius</u>- a segment that has one endpoint at the center and the other endpoint on the circle

<u>chord</u>- a segment that has its endpoints on the circle

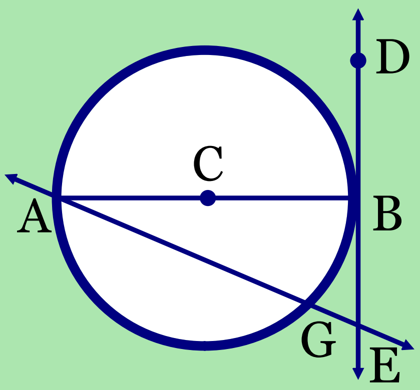
<u>diameter</u>- a chord that contains the center





*A line is **<u>tangent</u>** to a circle if it intersects the circle in exactly one point. This point is called the **point of tangency**.*

*A **<u>secant</u>** is a line that intersects the circle in two points.*



Example 1

Tell whether the line, ray, or segment is best described as a radius, chord, diameter, secant, or tangent of ⊙C.

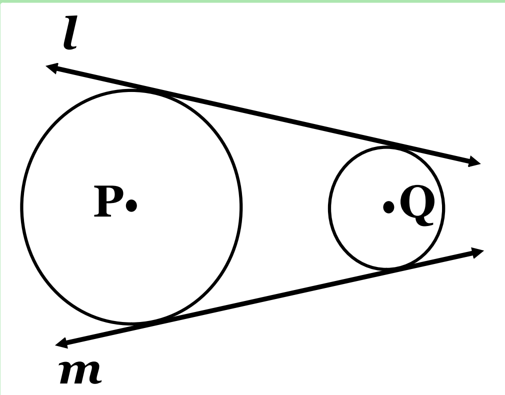
a) $\overline{AC}$ radius

b) $\overline{AB}$ diameter

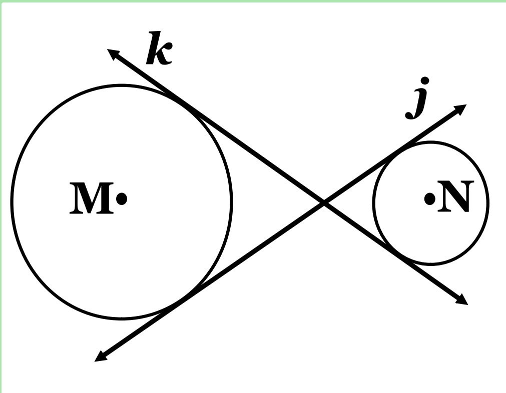
c) $\overrightarrow{DE}$ tangent

d) $\overleftrightarrow{AE}$ secant

A line, ray, or segment that is tangent to two coplanar circles is called a common tangent.



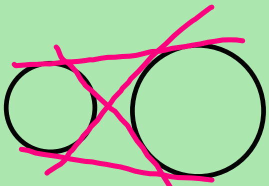
external tangents



internal tangents

Example 2

How many common tangents can the circles below have?

a)

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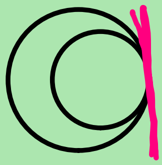
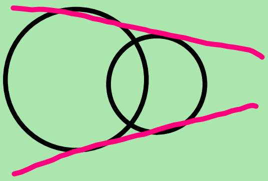
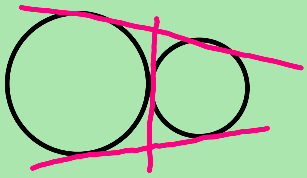
b)

1

c)

2

d)

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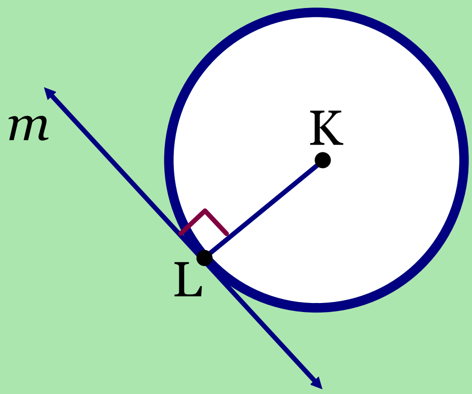
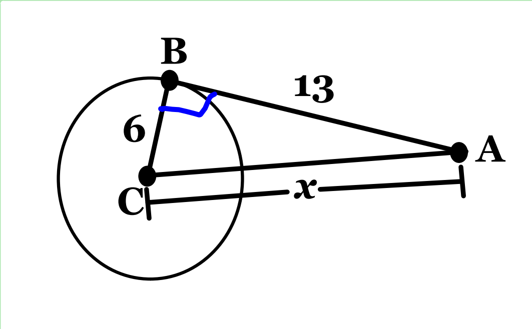
Theorems 10.1-10.2

A line is tangent to a circle if and only if the line is perpendicular to the radius of the circle drawn to the point of tangency.



Example 3

Refer to ⊙C with tangent $\overline{AB}$.
Find x as an exact answer.



$6^2 + 13^2 = x^2$

$36 + 169 = x^2$

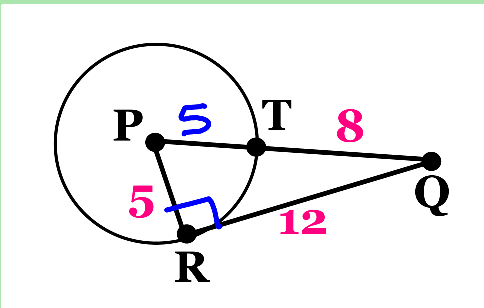
$205 = x^2$

$\boxed{\sqrt{205} = x}$

$5 \overline{)205}$
41

Example 4

Refer to ⊙P with radius $\overline{PR}$.
Show that QR is tangent to ⊙P.



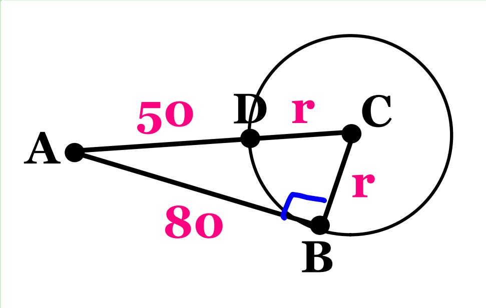
$5^2 + 12^2 \stackrel{?}{=} 13^2$

$25 + 144 = 169$

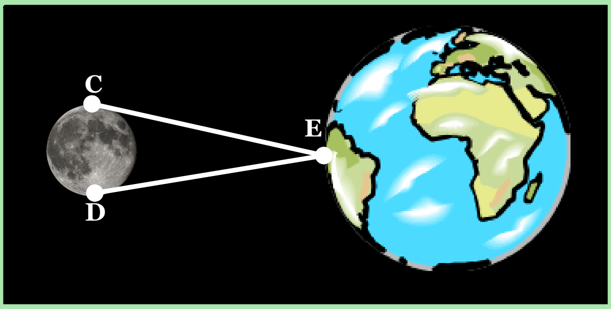
$169 = 169 \checkmark$

Example 5

Refer to $\odot C$ with B as the point of tangency. Find the radius of $\odot C$.



$$r^2 + 80^2 = (50 + r)^2$$
$$r^2 + 6400 = (50 + r)(50 + r)$$
$$\cancel{r^2} + 6400 = 2500 + 50r + 50r + \cancel{r^2}$$
$$-\cancel{r^2} \qquad\qquad -\cancel{r^2}$$
$$6400 = 2500 + 100r$$
$$-2500 \qquad -2500$$
$$\frac{3900}{100} = \frac{100r}{100}$$
$$\boxed{39 = r}$$



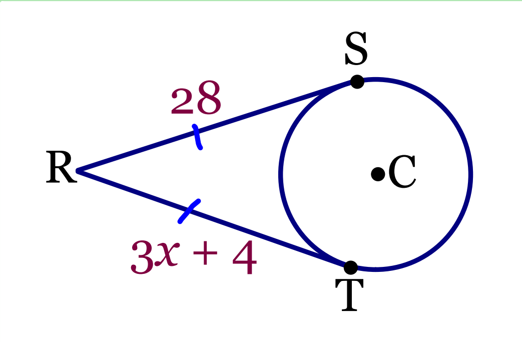
*$\overline{EC}$ and $\overline{ED}$ are examples of two **tangent segments** drawn from a common point E outside the circle.*

Theorem 10.3

If two segments from the same exterior point are tangent to a circle, then they are congruent.

Example 6

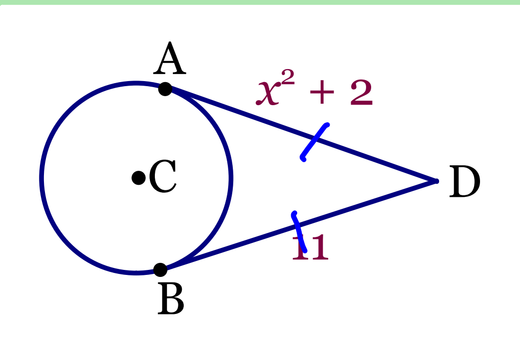
$\overline{RS}$ is tangent to ⊙C at S and $\overline{RT}$ is tangent to ⊙C at T. Find the value of x.



$$3x + 4 = 28$$
$$-4 \quad -4$$
$$\frac{3x}{3} = \frac{24}{3}$$
$$x = 8$$

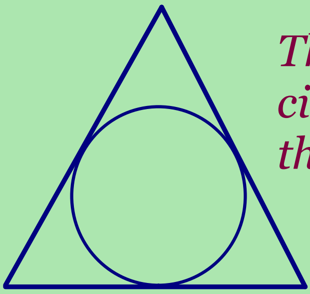
Example 7

$\overline{DA}$ is tangent to ⊙C at A and $\overline{DB}$ is tangent to ⊙C at B. Find the value of x.



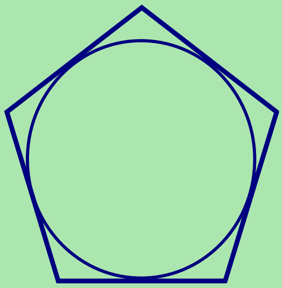
$$x^2 + 2 = 11$$
$$-2 \quad -2$$
$$\sqrt{x^2} = \sqrt{9}$$
$$\boxed{x = \pm 3}$$

A polygon is ***circumscribed*** *about a circle if each side of the polygon is tangent to the circle.*



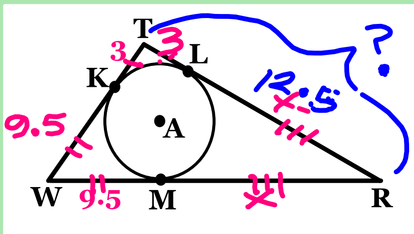
These polygons are circumscribed about the circles.

The circles are inscribed in the polygons.



Example 8

Triangle TRW is circumscribed about ⨀A. If the perimeter of △TRW is 50, TK = 3, and WM = 9.5, find TR.



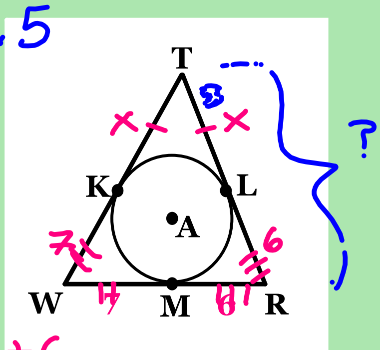
$$3+3+9.5+9.5+x+x = 50$$
$$25 + 2x = 50$$
$$2x = 25$$
$$x = 12.5$$

$TR = 3+12.5$

$\boxed{TR = 15.5}$

Example 9

Triangle TRW is circumscribed about ⨀A. If the perimeter of △TRW is 42, MR = 6, and WM = 7, find TR.



$$6+6+7+7+x+x=42$$
$$26 + 2x = 42$$
$$2x = 16$$
$$x = 8$$

$TR = 8+6$

$\boxed{TR = 14}$