7.3 Part 1 Double-Angle Formulas

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$(1-\sin^2 x)-\sin^2 x$ $\quad = 1 - 2\sin^2 x$

$\cos^2 x + -(1+\cos^2 x)$ $\quad = 2\cos^2 x - 1$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Proof of sin 2x

$\sin 2x = \sin(x + x)$

$= \sin x \cos x + \cos x \sin x$

$= 2 \sin x \cos x$

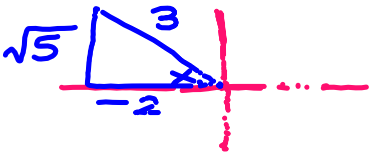
Now prove cos 2x

$\cos 2x = \cos(x+x)$

$= \cos x \cos x - \sin x \sin x$

$= \cos^2 x - \sin^2 x$

Example 1: If $\cos x = -\frac{2}{3}$ and x is in Quadrant II, find cos 2x, sin 2x, and tan 2x.



$\sin 2x = 2\sin x\cos x$

$= \frac{2}{1} \cdot \frac{\sqrt{5}}{3} \cdot -\frac{2}{3}$

$\boxed{\sin 2x = -\frac{4\sqrt{5}}{9}}$

$(-2)^2 + b^2 = (3)^2$

$4 + b^2 = 9$

$b^2 = 5$

$b = \sqrt{5}$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(-\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$

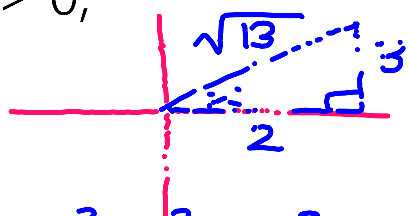
$= \frac{4}{9} - \frac{5}{9}$

$\boxed{\cos 2x = -\frac{1}{9}}$

$\tan 2x = \dfrac{-\frac{4\sqrt{5}}{9}}{\cancel{-\frac{1}{9}}} \cdot -\frac{9}{1}$

$\boxed{\tan 2x = 4\sqrt{5}}$

Example 2: If I III $\cot x = \frac{2}{3}$ and I II $\sin x > 0$, find sin 2x, cos 2x, and tan 2x.



$\sin 2x = 2\sin x\cos x$

$= 2 \cdot \frac{3}{\sqrt{13}} \cdot \frac{2}{\sqrt{13}}$

$\boxed{\sin 2x = \frac{12}{13}}$

$2^2 + 3^2 = c^2$

$13 = c^2$

$\cos 2x = \cos^2 x - \sin^2 x$

$= \left(\frac{2}{\sqrt{13}}\right)^2 - \left(\frac{3}{\sqrt{13}}\right)^2$

$= \frac{4}{13} - \frac{9}{13}$

$\boxed{\cos 2x = -\frac{5}{13}}$

$\tan 2x = \dfrac{\frac{12}{13}}{\cancel{-\frac{5}{13}}} \cdot -\frac{13}{5}$

$\boxed{\tan 2x = -\frac{12}{5}}$

Example 3: Write cos 3x in terms of cos x.

$$\cos 3x = \cos(2x + x)$$
$$= \cos 2x \cos x - \sin 2x \sin x$$
$$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \sin x$$
$$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$$
$$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$$
$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$
$$\cos 3x = 4\cos^3 x - 3\cos x$$

Example 4: Verify the identity.

$$\frac{\sin 3x}{\sin x \cos x} = 4 \cos x - \sec x$$

Example 5: Write the given expression as an algebraic expression in x.

$\sin(2\tan^{-1}x)$

$\tan^{-1}x = \theta$

$\tan\theta = \frac{x}{1}$

$x^2 + 1^2 = c^2$

$x^2 + 1 = c^2$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \cdot \frac{x}{\sqrt{x^2+1}} \cdot \frac{1}{\sqrt{x^2+1}}$

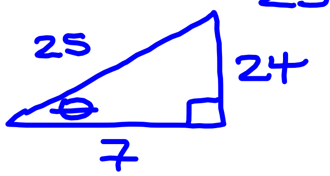
$= \frac{2x}{x^2+1}$

Example 6: Find the exact value of the given expression.

$\sin(2\cos^{-1}\frac{7}{25})$

$\cos^{-1}\frac{7}{25} = \theta$

$\cos\theta = \frac{7}{25}$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \cdot \frac{24}{25} \cdot \frac{7}{25}$

$= \frac{336}{625}$