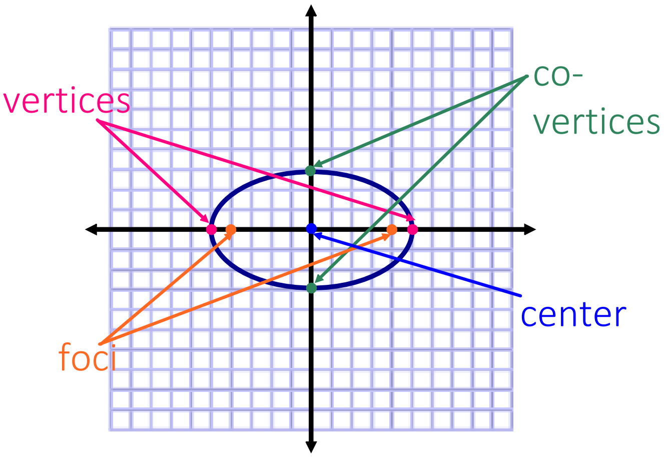
9.4 Part 1 Ellipses



a = distance from center to vertex

a^2 = larger denominator

b = distance from center to co-vertex

b^2 = smaller denominator

c = distance from center to a focus point

$c^2 = a^2 - b^2$

Standard Equation for a Horizontal Ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Center: (0,0)

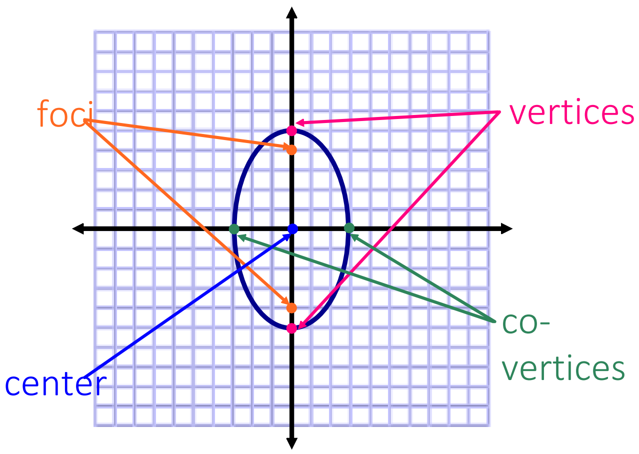
Vertices: (±a,0)

Co-Vertices: (0,±b)

Foci: (±c,0)

Major Axis: horizonal, length is 2a

Minor Axis: vertical, length is 2b



REMINDER: a^2 is always the larger denominator!! Whichever variable a is under will tell whether the ellipse is horizontal or vertical.

Standard Equation for a Vertical Ellipse

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Center: (0,0)

Vertices: (0,±a)

Co-Vertices: (±b,0)

Foci: (0,±c)

Major Axis: vertical, length is 2a

Minor Axis: horizontal, length is 2b

Example 1

Find the coordinates of the center, vertices, co-vertices, and foci.
Then sketch the graph.

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$a^2 = 25 \rightarrow a = 5$

$b^2 = 9 \rightarrow b = 3$

center $(0,0)$

vertices $(-5,0), (5,0)$

co-vert. $(0,-3), (0,3)$

$c^2 = a^2 - b^2$

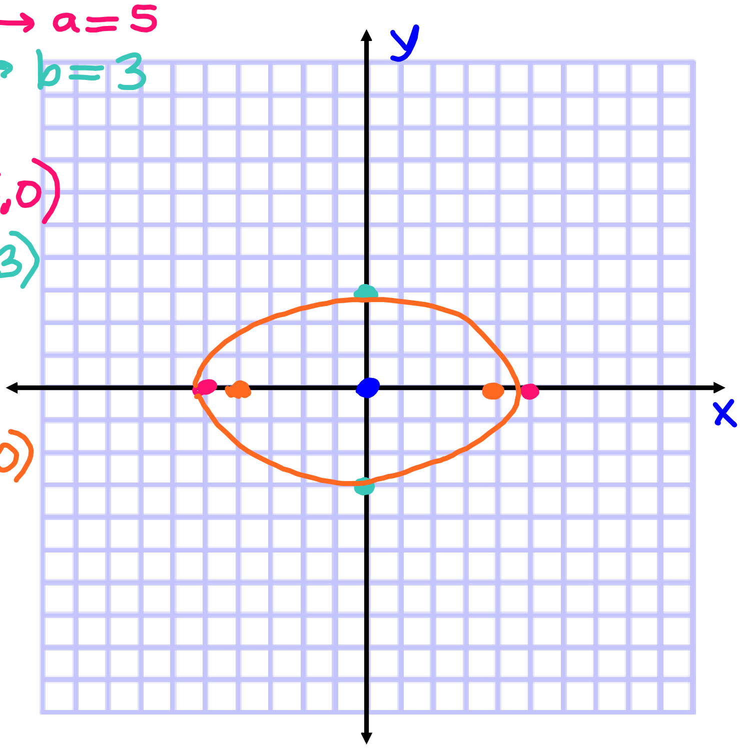
$c^2 = 25 - 9$

$c^2 = 16$

$c = 4$

foci

$(-4,0), (4,0)$



Example 2

Find the coordinates of the center, vertices, co-vertices, and foci.
Then sketch the graph.

$$\frac{x^2}{36} + \frac{y^2}{81} = 1$$

$a^2 = 81 \rightarrow a = 9$

$b^2 = 36 \rightarrow b = 6$

center $(0,0)$

Vertices $(0,-9), (0,9)$

co-vert $(-6,0), (6,0)$

$c^2 = a^2 - b^2$

$c^2 = 81 - 36$

$c^2 = 45$

$c = 3\sqrt{5}$

$3 \overline{)45}$

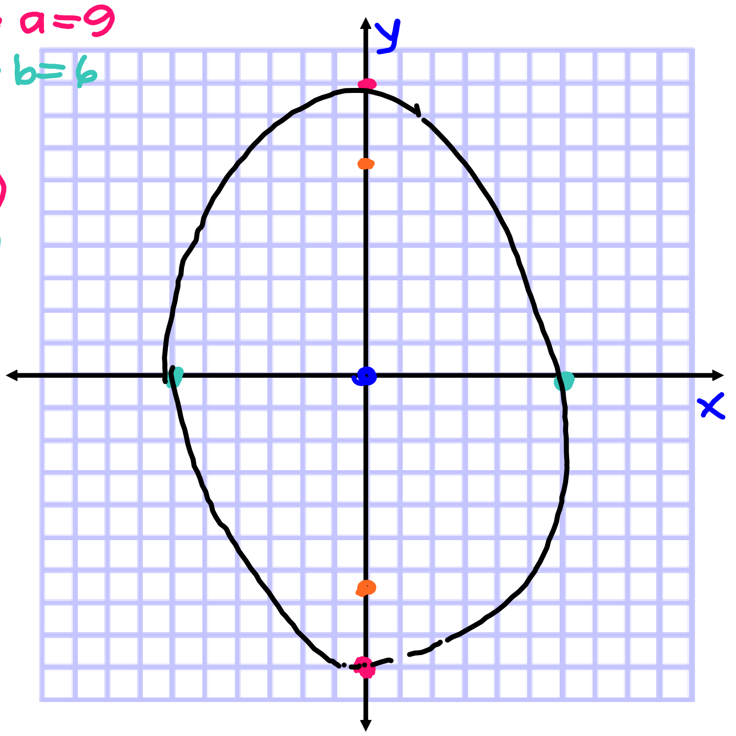
$3 \overline{)15}$

5

foci

$(0,-3\sqrt{5})$,

$(0,3\sqrt{5})$



Example 3

$a=10$ $c=6$

The vertices of an ellipse are (±10,0) and the foci are (±6,0). Find the equation and sketch the graph.

$a^2=100$ $c^2=36$

$c^2=a^2-b^2$

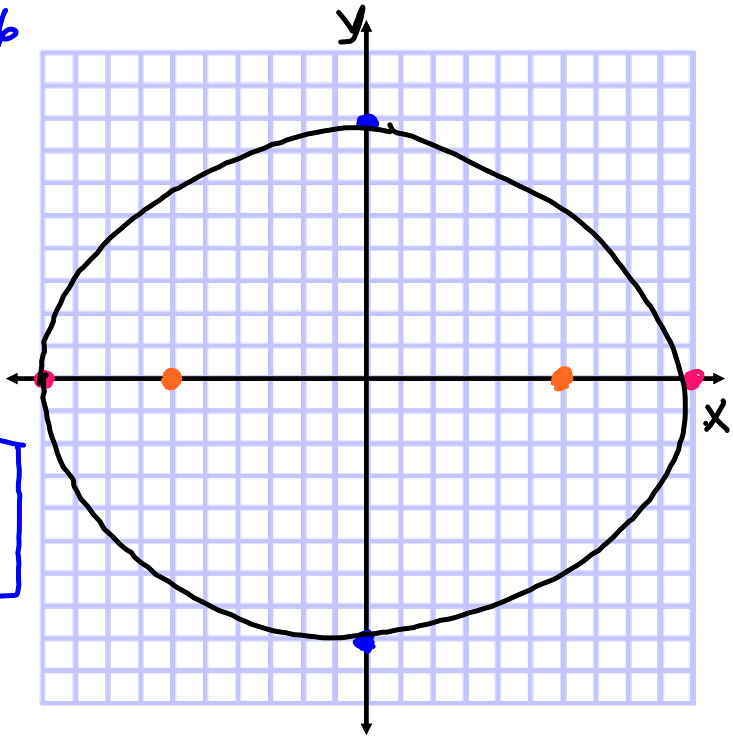
$36=100-b^2$

$-64=-b^2$

$64=b^2$

$8=b$

$$\boxed{\frac{x^2}{100}+\frac{y^2}{64}=1}$$



Example 4

Find the coordinates of the center, vertices, co-vertices, and foci. Then sketch the graph.

$16x^2 + 9y^2 = 144.$

$\frac{16x^2}{144} + \frac{9y^2}{144} = \frac{144}{144}$

$a^2=16 \rightarrow a=4$

$b^2=9 \rightarrow b=3$

$c^2=a^2-b^2$

$c^2=16-9$

$c^2=7 \rightarrow c=\sqrt{7}$

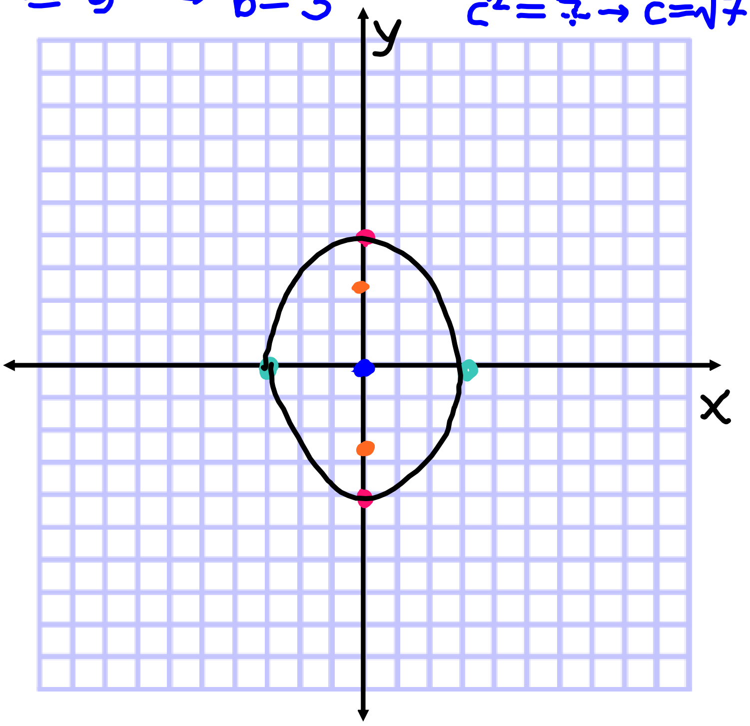
$\frac{x^2}{9}+\frac{y^2}{16}=1$

center (0,0)

vert (0,4), (0,-4)

co-vert (-3,0), (3,0)

foci $(0,\sqrt{7}), (0,-\sqrt{7})$



Standard Equation for a Translated Ellipse

Standard Equation for a Horizontal Ellipse	Standard Equation for a Vertical Ellipse
$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

Example 5

Find the center, vertices, co-vertices, and foci of the ellipse with the equation $5(x-2)^2 + 4(y+3)^2 = 20$.

$$\frac{5(x-2)^2}{20} + \frac{4(y+3)^2}{20} = \frac{20}{20}$$

$$\frac{(x-2)^2}{4} + \frac{(y+3)^2}{5} = 1$$

$a^2 = 5 \rightarrow a = \sqrt{5}$

$b^2 = 4 \rightarrow b = 2$

$c^2 = a^2 - b^2 \rightarrow c^2 = 5 - 4$

$c^2 = 1$

$c = 1$

center $(2, -3)$

vertices $(2, -3 \pm \sqrt{5})$

co-vert: $(0, -3), (4, -3)$

foci $(2, -2), (2, -4)$

Example 6

Find the coordinates of the center, vertices, co-vertices, and foci. Then sketch the graph.

$$\frac{(x+3)^2}{49} + \frac{(y-4)^2}{25} = 1$$

$a^2 = 49$

$a = 7$

$b^2 = 25$

$b = 5$

$c^2 = a^2 - b^2$

$c^2 = 49 - 25$

$c^2 = 24$

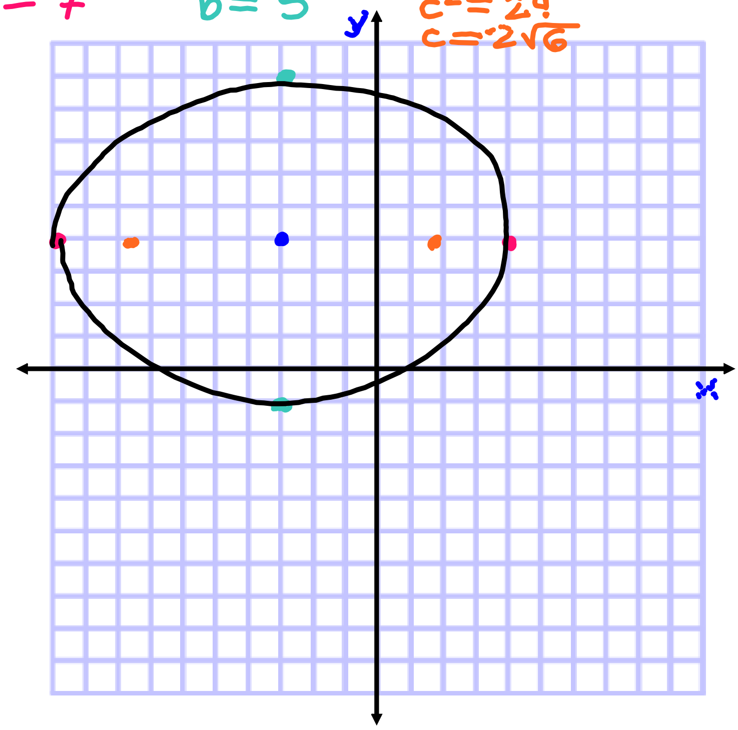
$c = 2\sqrt{6}$

center $(-3, 4)$

vert $(-10, 4), (4, 4)$

co-vert $(-3, 9), (-3, -1)$

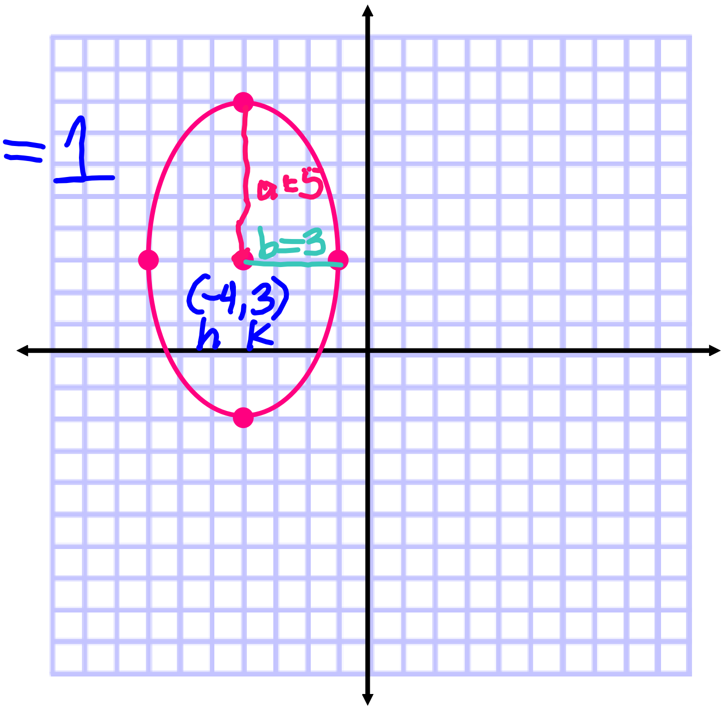
foci $(-3 \pm 2\sqrt{6}, 4)$



Example 7

Write the standard equation for the ellipse.

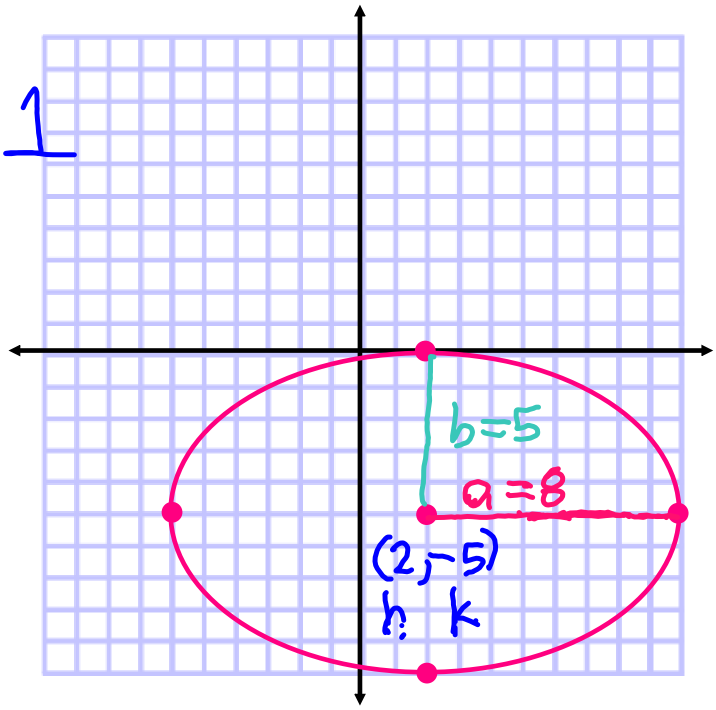
$$\frac{(x+4)^2}{9} + \frac{(y-3)^2}{25} = 1$$



Example 8

Write the standard equation for the ellipse.

$$\frac{(x-2)^2}{64} + \frac{(y+5)^2}{25} = 1$$



9.4 Part 2 Ellipses

Example 9

An ellipse is defined by $4x^2 + y^2 + 24x - 4y + 36 = 0$. Write the standard equation, and identify the coordinates of its center, vertices, co-vertices, and foci.

$$4x^2 + 24x \quad + y^2 - 4y \quad = -36$$

$$4(x^2 + 6x + 9) + y^2 - 4y + 4 = -36 + 36 + 4$$

$\frac{1}{2}(6) = 3$ $\quad\quad$ $\frac{1}{2}(-4) = -2$

$(3)^2 = 9$ $\quad\quad$ $(-2)^2 = 4$

$$\frac{4(x+3)^2}{4} + \frac{(y-2)^2}{4} = \frac{4}{4}$$

$$\frac{(x+3)^2}{1} + \frac{(y-2)^2}{4} = 1$$

$a^2 = 4 \longrightarrow a = 2$

$b^2 = 1 \longrightarrow b = 1$

$c^2 = 4 - 1$

$c^2 = 3 \longrightarrow c = \sqrt{3}$

center $(-3, 2)$

vertices $(-3, 4), (-3, 0)$

co-vert $(-4, 2), (-2, 2)$

foci $(-3, 2 \pm \sqrt{3})$

Example 10

An ellipse is defined by $9x^2 + 16y^2 - 36x - 64y - 44 = 0$. Write the standard equation, and identify the coordinates of its center, vertices, co-vertices, and foci.