7.2 Part 1: Addition and Subtraction Formulas

Formulas for Sine

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

Formulas for Cosine

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

Formulas for Tangent

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \qquad \tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Example 1: Find the exact value of each expression.

a) $\cos 75^\circ$ $\cos(45^\circ + 30^\circ)$

$$\cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

$\frac{\cancel{\pi}}{12} \cdot \frac{180}{\cancel{\pi}} = 15^\circ$

b) $\cos \frac{\pi}{12}$

$\cos 15^\circ \longrightarrow \cos(45^\circ - 30^\circ)$

$$\cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$$

$$\boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$$

Example 2: Find the exact value of each expression.

a) $\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$

$\sin(20^\circ + 40^\circ)$

$\sin 60^\circ$

$\boxed{\frac{\sqrt{3}}{2}}$

b) $$\frac{\tan\frac{\pi}{18} + \tan\frac{\pi}{9}}{1 - \tan\frac{\pi}{18}\tan\frac{\pi}{9}}$$

$\tan\left(\frac{\pi}{18} + \frac{\pi}{9}\right)$

$\tan\left(\frac{\pi}{18} + \frac{2\pi}{18}\right)$

$\tan\frac{3\pi}{18}$

$\tan\frac{\pi}{6}$

$\boxed{\frac{\sqrt{3}}{3}}$

Example 3: Verify the identity.

$\cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\cos\frac{\pi}{2}\cos u + \sin\frac{\pi}{2}\sin u =$

$\cancel{0 \cdot \cos u} + 1 \cdot \sin u =$

$\sin u = \sin u$ ✓

Example 4: Verify the identity.

$$\frac{1 + \tan x}{1 - \tan x} = \tan\left(\frac{\pi}{4} + x\right)$$

$$= \frac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}$$

$$\checkmark \frac{1+\tan x}{1-\tan x} = \frac{1+\tan x}{1-\tan x}$$

An identity from Calculus...

Example 5: If f(x) = sin x, show that

$$\frac{f(x+h) - f(x)}{h} = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right)$$

$$\frac{\sin(x+h) - \sin x}{h} =$$

$$\frac{\sin x\cos h + \cos x\sin h - \sin x}{h} =$$

$$\frac{\sin x\cos h - \sin x}{h} + \frac{\cos x\sin h}{h} =$$

$$\sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) = \sin x\left(\frac{\cos h - 1}{h}\right) + \cos x\left(\frac{\sin h}{h}\right) \checkmark$$

7.2 Part 2: Addition and Subtraction Formulas

Example 1: Verify the identity.

$\cos(x - \frac{\pi}{2}) = \sin x$

Example 2: Verify the identity.

$\sin(\frac{\pi}{2} - x) = \sin(\frac{\pi}{2} + x)$

Example 3: Verify the identity.

$$\tan(x - \frac{\pi}{4}) = \frac{\tan x - 1}{\tan x + 1}$$

Example 4: Verify the identity.

$$\sin(x + y) - \sin(x - y) = 2\cos x \sin y$$

$$(\cancel{\sin x \cos y} + \cos x \sin y) + -(\cancel{\sin x \cos y} - \cos x \sin y) =$$

$$2 \cos x \sin y = 2 \cos x \sin y \checkmark$$

Example 5: Verify the identity.

$$\cot(x + y) = \frac{\cot x \cot y - 1}{\cot x + \cot y}$$

Example 6: Verify the identity.

$$1 - \tan x \tan y = \frac{\cos(x + y)}{\cos x \cos y}$$

$$= \frac{\cos x \cos y - \sin x \sin y}{\cos x \cos y}$$

$$= \cancel{\frac{\cos x \cos y}{\cos x \cos y}} - \frac{\sin x \sin y}{\cos x \cos y}$$

$$\checkmark \; 1 - \tan x \tan y = 1 - \tan x \tan y$$

Example 7: Verify the identity.

$$\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$$