

7.1 Part 3 Trigonometric Identities

Example 1: Verify the identity.

$$(\sin \alpha - \tan \alpha)(\cos \alpha - \cot \alpha) = (\cos \alpha - 1)(\sin \alpha - 1)$$

$$\left(\sin \alpha - \frac{\sin \alpha}{\cos \alpha}\right)\left(\cos \alpha - \frac{\cos \alpha}{\sin \alpha}\right) =$$

$$\sin \alpha \cos \alpha - \cancel{\sin \alpha} \cdot \frac{\cos \alpha}{\cancel{\sin \alpha}} - \frac{\sin \alpha}{\cancel{\cos \alpha}} \cdot \cancel{\cos \alpha} + \frac{\sin \alpha}{\cancel{\cos \alpha}} \cdot \frac{\cancel{\cos \alpha}}{\cancel{\sin \alpha}}$$

$$(\sin \alpha \cos \alpha - \cos \alpha) - (\sin \alpha + 1) =$$

$$\cos \alpha (\cancel{\sin \alpha - 1}) - 1 (\cancel{\sin \alpha - 1}) =$$

$$(\sin \alpha - 1)(\cos \alpha - 1) = (\cos \alpha - 1)(\sin \alpha - 1) \checkmark$$

Example 2: Verify the identity.

$$\frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

$$\frac{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}}{\frac{\cos x}{\sin x} + \frac{\cos y}{\sin y}} =$$

$$\frac{\frac{\sin x \cos y}{\cos x \cos y} + \frac{\sin y \cos x}{\cos y \cos x}}{\frac{\cos x \sin y}{\sin x \sin y} + \frac{\cos y \sin x}{\sin y \sin x}} =$$

$$\frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} =$$

$$\frac{\cos x \sin y + \sin x \cos y}{\sin x \sin y}$$

$$\frac{\cancel{\sin x \cos y} + \cancel{\cos x \sin y}}{\cos x \cos y} \cdot \frac{\sin x \sin y}{\cancel{\cos x \sin y} + \cancel{\sin x \cos y}} =$$

$$\tan x \tan y = \tan x \tan y \checkmark$$

Example 3: Verify the identity.

$$\frac{\tan v - \cot v}{\tan^2 v - \cot^2 v} = \sin v \cos v$$

$$\frac{\cancel{\tan v - \cot v}}{(\tan v + \cot v)(\cancel{\tan v - \cot v})} =$$

$$\frac{1}{\frac{\sin v}{\cos v} + \frac{\cos v}{\sin v}} =$$

$$\frac{1}{\frac{\sin^2 v + \cos^2 v}{\cos v \sin v}} =$$

$$1 \cdot \cos v \sin v = \sin v \cos v \quad \checkmark$$

Example 4: Verify the identity.

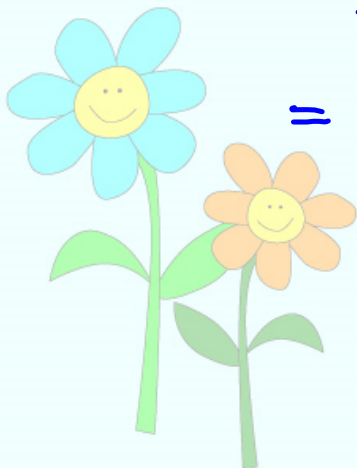
$$\frac{\cot x + 1}{\cot x - 1} = \frac{1 + \tan x}{1 - \tan x}$$

$$= \frac{\frac{\cancel{\cos x}}{\cancel{\cos x}} + \frac{\sin x}{\cos x}}{\frac{\cancel{\cos x}}{\cancel{\cos x}} - \frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}$$

$$= \frac{\cancel{\cos x} + \sin x}{\cancel{\cos x}} \cdot \frac{\cancel{\cos x}}{\cos x - \sin x}$$

cancel
while
down



Example 5: Verify the identity.

$$\tan^2 x - \cot^2 x = \sec^2 x - \csc^2 x$$

$$(\tan x + \cot x)(\tan x - \cot x) =$$

$$\left(\frac{\sin^2 x}{\cos x \cdot \sin x} + \frac{\cos^2 x}{\sin x \cdot \cos x} \right) \left(\frac{\sin^2 x}{\cos x \cdot \sin x} - \frac{\cos^2 x}{\sin x \cdot \cos x} \right) =$$

$$\left(\frac{1}{\cos x \sin x} \right) \left(\frac{\sin^2 x - \cos^2 x}{\cos x \sin x} \right) =$$

$$\frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x} =$$

$$\frac{\cancel{\sin^2 x}}{\cos^2 x \cancel{\sin^2 x}} - \frac{\cancel{\cos^2 x}}{\cos^2 x \cancel{\sin^2 x}} =$$

$$\sec^2 x - \csc^2 x = \sec^2 x - \csc^2 x \quad \checkmark$$

Example 6: Verify the identity.

$$\csc x - \sin x = \cos x \cot x$$

$$= \cos x \cdot \frac{\cos x}{\sin x}$$

$$= \frac{\cos^2 x}{\sin x}$$

$$= \frac{1 - \sin^2 x}{\sin x}$$

$$= \frac{1}{\sin x} - \frac{\cancel{\sin^2 x}}{\cancel{\sin x}}$$

$$\csc x - \sin x = \csc x - \sin x \quad \checkmark$$