7.1 Part 3 Trigonometric Identities

Example 1: Verify the identity.

$(\sin\alpha - \tan\alpha)(\cos\alpha - \cot\alpha) = (\cos\alpha - 1)(\sin\alpha - 1)$

$\left(\sin\alpha - \frac{\sin\alpha}{\cos\alpha}\right)\left(\cos\alpha - \frac{\cos\alpha}{\sin\alpha}\right) =$

$\sin\alpha\cos\alpha - \sin\alpha\cdot\frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha}\cdot\cos\alpha + \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\cos\alpha}{\sin\alpha}$

$\sin\alpha\cos\alpha - \cos\alpha - \sin\alpha + 1 =$

$\cos\alpha(\sin\alpha - 1) - 1(\sin\alpha - 1) =$

$(\sin\alpha - 1)(\cos\alpha - 1) = (\cos\alpha - 1)(\sin\alpha - 1)$ ✓

Example 2: Verify the identity.

$$\frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

$$\frac{\frac{\sin x}{\cos x}\cdot\frac{\cos y}{\cos y} + \frac{\sin y}{\cos y}\cdot\frac{\cos x}{\cos x}}{\frac{\cos x}{\sin x}\cdot\frac{\sin y}{\sin y} + \frac{\cos y}{\sin y}\cdot\frac{\sin x}{\sin x}} =$$

$$\frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y} =$$

$$\frac{\cos x\sin y + \sin x\cos y}{\sin x\sin y}$$

$$\frac{\sin x\cos y + \cos x\sin y}{\cos x\cos y}\cdot\frac{\sin x\sin y}{\cos x\sin y + \sin x\cos y} =$$

$\tan x \tan y = \tan x \tan y$ ✓

Example 3: Verify the identity.

$$\frac{\tan v - \cot v}{\tan^2 v - \cot^2 v} = \sin v \cos v$$

$$\frac{\tan v - \cot v}{(\tan v + \cot v)(\tan v - \cot v)} =$$

$$\frac{1}{\frac{\sin^2 v}{\cos v \sin v} + \frac{\cos^2 v}{\sin v \cos v}} =$$

$$\frac{1}{\frac{\sin^2 v + \cos^2 v}{\cos v \sin v}} =$$

$$1 \cdot \cos v \sin v = \sin v \cos v \checkmark$$

Example 4: Verify the identity.

$$\frac{\cot x + 1}{\cot x - 1} = \frac{1 + \tan x}{1 - \tan x}$$

$$= \frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}$$

$$= \frac{\frac{\cos x + \sin x}{\cos x}}{\frac{\cos x - \sin x}{\cos x}}$$

$$= \frac{\cos x + \sin x}{\cos x} \cdot \frac{\cos x}{\cos x - \sin x}$$

Work on white board.

Example 5: Verify the identity.

$\tan^2 x - \cot^2 x = \sec^2 x - \csc^2 x$

$(\tan x + \cot x)(\tan x - \cot x) =$

$\left(\frac{\sin^2 x}{\cos x \cdot \sin x} + \frac{\cos^2 x}{\sin x \cdot \cos x}\right)\left(\frac{\sin^2 x}{\cos x \cdot \sin x} - \frac{\cos^2 x}{\sin x \cdot \cos x}\right) =$

$\left(\frac{1}{\cos x \sin x}\right)\left(\frac{\sin^2 x - \cos^2 x}{\cos x \sin x}\right) =$

$\frac{\sin^2 x - \cos^2 x}{\cos^2 x \sin^2 x} =$

$\frac{\sin^2 x}{\cos^2 x \sin^2 x} - \frac{\cos^2 x}{\cos^2 x \sin^2 x} =$

$\sec^2 x - \csc^2 x = \sec^2 x - \csc^2 x$!

Example 6: Verify the identity.

$\csc x - \sin x = \cos x \cot x$

$= \cos x \cdot \frac{\cos x}{\sin x}$

$= \frac{\cos^2 x}{\sin x}$

$= \frac{1 - \sin^2 x}{\sin x}$

$= \frac{1}{\sin x} - \frac{\sin^2 x}{\sin x}$

$\csc x - \sin x = \csc x - \sin x$ ✓